Working Papers Nos. 38 and 39

38 : **The Special Bearer Bonds Scheme : Pedigree and Prospects**

by

K. Bharathan

39 : **Some Implications of the Special Bearer Bonds Scheme : an Application of Elementary Supply-Demand Analysis**

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by

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*I owe a great deal to discussions with K.Bharathan, S.Majumdar and K.Nagaraj; also much to Dilip Mookherjee, to whose several comments and criticisms I have managed to do scant justice. All surviving errors are entirely my own.

Some Implications of the Special Bearer Bonds-Scheme : An Application of Elementary Supply-Demand Analysis

Introduction

This paper seeks to examine some implications of the Special Bearer Bonds Scheme recently introduced by the Government as one more in a series of efforts of questionable success to reduce the quantum of black money circulating in the economy. The analysis has been done within the framework of an extremely simple supply-demand model, which essentially represents an attempt at formalizing a part of the analysis presented by K.Bharathan (1983).[1]

The main conclusion of the paper is the somewhat self-evident one that the success of the Scheme would depend crucially on how seriously black-marketeers will take the Government's professed intent to mop up black-money in the system; and further, that even if one should allow for considerable fear among black-marketeers of being raided by the Income-Tax Authorities, any initial success in terms of sale of bonds by the Government could *possibly* be vitiated by off-setting increases in the circulation of black money due to the development of a demand for bonds among *white* money holders. The details are spelt out in the text of the paper.

The essential features of the Special Bearer Bonds Scheme can be summarized as follows: (See Bharathan, *op.cit.*).

(1) The issue-price of each bond is Rs.10,000/-.

(2) The bonds are redeemable in ten years' time, i.e., in 1991 -- at which time the bearer will be entitled to the issue-price of the bond less income-tax thereon, together with a bonus of Rs.2,000/-.

(3) The anonymity of the purchaser of the bond will be preserved.

(4) The bonds can serve as security against loans raised from a commercial bank.

(5) The bond will not be taken into account in assessing the wealth-tax of the owner, nor will it attract gift-tax.

(6) Knowledge of the possession of the bearer-bond will not be used to initiate any enquiry against the bearer.

(7) There is no restriction on the number of bonds that can be purchased by any one individual.

In what follows, it is posited that there will be two categories of demanders of bonds: the first is constituted by black-money holders (this category can be called the 'black demanders') and the second by white-money holders (this category can be called the 'white demanders'). The underlying logic of the ensuing exercise can be compactly summarized as follows. All black-demanders who find it worthwhile to trade their black assets for bonds (and this would depend, inter alia, on their assessment of the risks involved in holding on to black assets) will purchase bonds at the time of their issue. At the time of closure of issue of bonds, black demanders will have excess (in the sense of 'unsatisfied') demands for bonds if they are in possession of black assets which could not be liquidated to purchase bonds at the time of their issue. These excess demands will trigger off a market for bonds after the closure of their issue. The equilibrium price of bonds in the post-issue market will, under some assumptions, increase with time, and under other assumptions, decline with time. Meanwhile, the white-asset holders will also enter into the market for bonds with <u>their</u> demand-prices (which could depend, <u>inter alia</u>, on their tax-liabilities, the going rate of return on white assets and the number of years left to maturity of the bond). If the equilibrium price of bonds as determined by the activities of the black-demanders were to decline over time, then this would be particularly conducive to the purchase of bonds by white demanders -- for under these circumstances, the chances of the demand-prices of white-demanders exceeding the prevailing

price of bonds would be enhanced. The ensuing exchange of white money for bonds would then be absorbed into the 'black' hoards; in varying measures (depending on various parametric value-assignments) the initial absorption by Government of black money through the sale of Bearer Bonds would tend to be neutralized; in principle, it is even possible that the scheme could generate a net increase in the circulation of black money.

The paper is divided into six sections. The first section deals with the determination of the equilibrium quantity of bonds purchased by black asset-holders at the time of issue of the bonds. The second section deals with the determination of the equilibrium price of bonds in the market for bonds that develops among black demanders immediately after the closure of issue of bonds. The third section is concerned with an examination of the behaviour of this equilibrium price over time. The fourth section introduces the white demanders of bonds and seeks to examine their demand-for-bonds function. The fifth section considers the effect of the activities of the white demanders on the equilibrium price as determined by black demanders. The sixth section examines the assumptions used in the model.

The issues involved in this note are of some interest to the theorist; the problem under review lends itself to modelling within a context in which time, uncertainty and a capital market with financial assets have a central role to play. The present paper is, however, very restricted and modest in its objectives. In particular, the note addresses itself to a very partial equilibrium approach to the problem, confining itself, as it does, to a consideration of the price-quantity implications over time of the Scheme for only the financial sector of the economy. The model presented in this note could perhaps be considerably enriched by inclusion of a real sector, which could be represented by one composite consumption commodity; it would then be interesting to examine the consequences of the Bonds scheme not only for allocation of portfolios among financial

assets, but also for allocation between financial assets on the one hand and consumption on the other. A major limitation of this note is that the latter exercise is not here undertaken; it is important to note that the current contribution goes nowhere near the presentation of a full scale macro theoretic general equilibrium analysis of the Black Economy.

With these preliminary remarks, I turn now to some formal aspects of the note.

I. The Determination of the Equilibrium Quantity of Bonds Demanded by Black Demanders at the Time of Issue of Bonds

Some preliminary definitions relating to the terminology employed in the note are in order. Let N be the set of black-asset holders, who shall be designated by the running index $j = 1, 2, \ldots, n$. Also define the following:

X ..... issue price of the bond;

m_j..... marginal rate of income-taxation, relevant to the income of individual j;

R ..... bonus payable to the bond-holder at the time of maturity of the bond;

t ..... variable designating time. Some specific values assumed by t are defined below:

t=o ... time of issue of bonds.

$t = -v_j$ time (prior to t=o) when individual j began operating in the black-market;

t=T ... time at which the bond matures, measured from t=o.

$t=L_j$... time-span of economic relevance to individual j's decision making ($L_j \geq T$).

A_j ..... stock of black assets possessed by individual j at the time of issue of bonds, of which

C_j ..... stock of black assets which is readily convertible (into liquid assets), and

B_j ..... stock of black assets which is non-convertible, so that $A_j \equiv C_j + B_j$;

$S^d_{t,j}$ ..... state of nature signifying discovery of black assets in individual j's possession at time t;

$S^u_{t,j}$ ..... state of nature signifying non-discovery of black assets in individual j's possession at time t;

$P^u_{t,j}$ ..... probability, in individual j's estimate, of being undiscovered in possession of black assets in time t, this probability being conditional on not having been discovered in all preceding periods, so that

$$P^u_{t,j} \equiv Prob\left(S^u_{t,j} \Big/ \bigcap_{\tau\in(-v_j,t-1)} S^u_{\tau,j}\right) = Prob\left(\bigcap_{\tau\in(-v_j,t)} S^u_{\tau,j}\right) \div Prob\left(\bigcap_{\tau\in(-v_j,t-1)} S^u_{\tau,j}\right)$$

$$\equiv (\text{say})\ P^{u'}_{t,j} \big/ P^{u'}_{t-1,j} \equiv (\text{say})\ P_j (\leq 1) \quad ; \quad t \in [-v_j, L_j).$$

(Note: We are assuming above, for reasons of analytical simplicity, that at any given point of time the joint probability of having been undiscovered in all preceding time-periods upto and including the point in time under consideration, declines at a constant proportinate rate of $|P_j - 1|$ per period with time);

$P^d_{t,j} (\equiv 1 - P^u_{t,j})(\equiv 1 - P_j)$ ..... probability, in individual j's estimate, of being discovered in possession of black assets at time t, this probability being conditional on not having been discovered in all preceding periods,

r ..... rate of return, per annum, on holding a rupee's worth of black assets;

ρ ..... rate of return, per annum, on holding a rupee's worth of white assets;

λ ..... marginal penal rate of taxation on black assets possessed by individual j, and applicable in the event of these assets being discovered in his possession.

Each individual will now be assumed to resolve the problem of choosing between retaining his black assets <u>as</u> black assets, on the one hand, and converting these to bonds, on the other, according to the criterion of which alternative entails a larger expected terminal value.

Making now the convenient assumption that bonds are continuously divisible, we can write the terminal value, for individual j, of investment of his entire black assets in bonds as :

$$(1) \quad V_j = \frac{A_j}{X}\{x(1-m_j)+R\}\exp(\rho(1-m_j)(L_j-T)) \quad , \quad j \in N$$

(Note that the expression $\frac{A_j}{X}\{x(1-m_j)+R\}$ in (1) above denotes the amount due to individual j when his bonds mature at t = T, and this amount will compound thereafter at an annual (after-tax) rate of return of $\rho(1-m_j)$ for a period of L_j-T years).

The computation of the expected terminal value, if j holds his entire black assets <u>as</u> black assets, is slightly more complicated, and is assisted by use of a 'payoff' tree. The 'payoff' tree drawn in Figure 1 below shows the payoff (in terminal value) and probability of payoff for each of the two states of nature (those of being discovered and not discovered in possession of black assets) that can occur in every time period. The 'payoff'-tree is drawn, for illustrative purposes, for only three time periods: t =0,1,2.

The upper of the two nodes in each time period represents the event of being discovered ($S^{d}_{t,j}$), while the lower node represents the event of not being discovered $(S^{u}_{t,j})$.

Figure 1 : Individual j's decision-tree

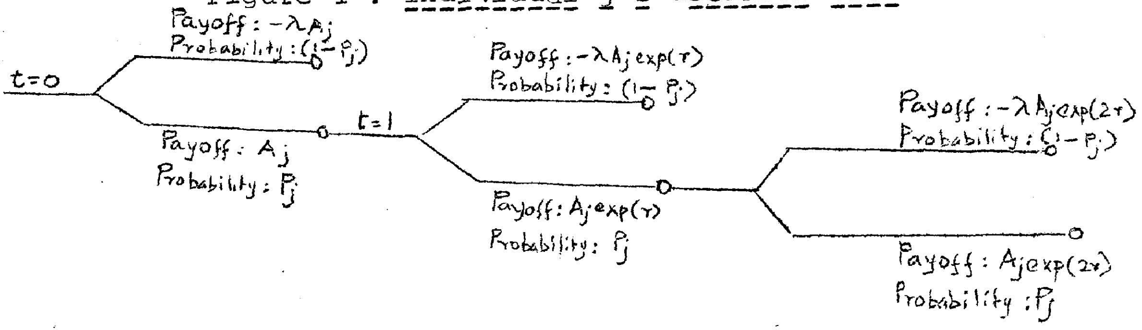


Figure 1 makes it clear that the terminal value W_j, if j holds his entire black assets as black assets, is a random variable with the following profile of payoffs and associated probabilities:

Payoff	Probability
$-\lambda A_j$	$1-P_j$
$-\lambda A_j \exp(r)$	$1-P_j$
$-\lambda A_j \exp(2r)$	$1-P_j$
$\vdots$	$\vdots$
$-\lambda A_j \exp(rL_j)$	$1-P_j$
$A_j \exp(rL_j)$	P_j

Given the profile above, it is immediately clear that the expected terminal value to j, if his entire black assets are held as black assets, is given by

$$\bar{W}_j = P_j A_j \exp(rL_j) - (1-P_j)\lambda A_j \int_0^{L_j} \exp(rt)\,dt \quad ; \quad j \in N$$

which, it may be verified, can be written as:[2/]

$$(2) \quad \bar{W}_j = \frac{A_j}{r}\left[P_j\{r\exp(rL_j)+\lambda\exp(rL_j)-\lambda\} - \lambda\{\exp(rL_j)-1\}\right] ; \quad j \in N$$

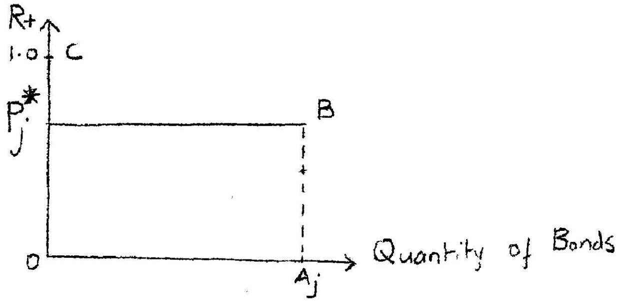
In line with the assumption that the choice between bonds and black assets is determined by the expected terminal values entailed by each option, one can say that individual j will

hold all his black assets as black assets if $V_j < \overline{W}_j$ and all his assets as bonds if $V_j \geq \overline{W}_j$. From (1) and (2), and after suitable manipulation, one can obtain the value of P_j for which V_j and $\overline{W}_j$ are equal as

$$(3) \quad P_j^* = \left(\frac{r}{X}\right) \frac{\left[\{X(1-m_j)+R\}\exp(\rho(1-m_j)(L_j-T)\}+\lambda\{\exp(rL_j)-1\}\right]}{\left[(r+\lambda)\exp(rL_j)-\lambda\right]} \; ; \; j \in \hat{N}$$

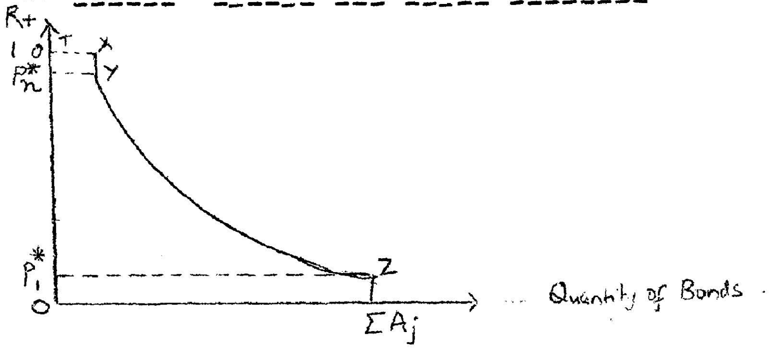
P_j^* may be called the 'critical subjective probability estimate' of individual j. If $P_j > P_j^*$, j will hold all his black assets as black assets, and if $P_j \leq P_j^*$, he will hold all his black assets in the form of bonds. We can plot j's 'demand for bonds' function as the curve $CP_j^* B A_j$ in Figure 2. (Note that this is not a conventional demand function with price on the vertical axis; rather, here, demand is expressed as a function of the subjective probability of being undiscovered in possession of black assets).

Figure 2: Individual j's 'demand-for-bonds' function



The horizontal aggregation of all n individuals' 'demand functions' will yield a step function which for large n can be approximated by the smooth downward sloping curve $TXYZ\Sigma A_j$ drawn in Figure 3. In plotting the curve in Figure 3, we have indexed the individuals in descending order of their 'critical subjective probability estimates', so that $P_n^* > P_{n-1}^* > \cdots P_1^*$.

Figure 3: Market 'Demand-for-Bonds-Function'



Note now that if $P_{min} \equiv \min_j \{P_j\}_{j \in N}$ and $P_{max} \equiv \max_j \{P_j\}_{j \in N}$, then it is clear straightaway that for $P_{min} > P^*_N$, the total quantity of bonds demanded will be zero (so that the Bonds Scheme must be judged an unqualified failure) and for $P_{max} \leq P^*_1$, the entire community of black-marketeers will hold their black assets in the form of bonds (so that the aggregate quantity of bonds demanded will be given by $D = \sum_1^n A_j$). These are the two polar cases. To be fully general, though, the curve in Figure 3 can be used to identify the quantity of bonds that will be demanded by resorting to the following simple iterative procedure.

Let q^1 be the largest integer for which $P^*_{q^1} \leq P_{min}$. Then the total quantity of bonds demanded by the first q^1 individuals will be given by $D^{q^1} = \sum_1^{q^1} A_j$. Let r^1 be the smallest integer for which $P^*_{r^1} \geq P_{max}$. It follows then that the quantity of bonds demanded by the jth individual will be given by $D_j = 0, j = r^1+1, r^1+2, \ldots, n$. Now compare q^1 and r^1. If $q^1 = r^1$, then the aggregate quantity of bonds demanded is simply D^{q^1}; if $q^1 \neq r^1$, then proceed as follows. Let $P'_{min} \equiv \min_j \{P_j\}_{j \in (q^1+1, r^1)}$ and $P'_{max} \equiv \max_j \{P_j\}_{j \in (q^1+1, r^1)}$. Let q^2 be the largest integer for which $P^*_{q^2} \leq P'_{min}$. Then the total quantity of bonds demanded by the first q^2 individuals is given by $D^{q^2} = \sum_1^{q^2} A_j$. Let r^2 be the smallest

integer for which $P^*_{r^2} \geq P'_{max}$. Then $D_j = 0$, $j = r^2+1, r^2+2, \ldots, r^1$. Compare q^2 and r^2. If $q^2 = r^2$, then repeat the entire procedure outlined above and continue to do so until in, say, the kth iteration, q^k and r^k converge. The aggregate quantity of bonds demanded will therefore be given as $D = \sum_1^{q^k(=r^k)} A_j$.

Now it may be imagined that the effective demand for bonds will be constrained by the quantity of convertible black assets an individual possesses. Without loss of generality, assume that for each $j \in (1, q^k)$, it is true that $C_j < A_j$. The effective quantity of bonds demanded by individual j will then be given by $D'_j = C_j$, so that, at the time of closure of issue of bonds, each of the q^k individuals will have unfulfilled demand to the extent of $A_j - C_j \equiv B_j$. Briefly, at the time of closure of issue of bonds, aggregate effective quantity of bonds demanded will be $D' = \sum_1^{q^k} C_j$, and aggregate unfulfilled demand will be $B = \sum_1^{q^k} B_j$. From hereonin, for notational convenience, we shall denote q^k by n^*.

II. The Determination of the Equlibrium Price in the Immediately 'Post-Issue' Market for Bonds

We can now turn our attention to the market for bonds that might be expected to develop among black demanders immediately after the issue of bonds has terminated. It will be assumed here that non-convertible black assets do not impede the purchase of bonds -- or more explicitly that exchange will be governed by barter. The focus in this section will be on deriving the market supply and demand functions for bonds. Consider first the demand side of the market. We begin by asking the question : What is the maximum

price which individual j will be willing to pay for a bond at the time (t=0) of closure of issue of bonds? Call this price $X_j^D(0)$. Then clearly $X_j^D(0)$ is that price which, given j's subjective probability estimate of being discovered in possession of black-assets, will ensure that the expected terminal value of investment in black assets worth this price will just equal the terminal value of holding on to a bond. The former is given by $\frac{X_j^D(0)}{r}\left[P_j\{r\exp(rL_j)+\lambda\exp(rL_j)-\lambda\}-\lambda\{\exp(rL_j)-1\}\right]$, and the latter by $\{X(1-m_j)+R\}\exp(\rho(1-m_j)(L_j-T))$.

By setting the above two quantities equal to one another, we can solve for $X_j^D(0)$, to obtain[3/]

$$X_j^D(0) = \left(\frac{r}{X}\right)\frac{\left[\{X(1-m_j)+R\}\exp(\rho(1-m_j)(L_j-T))\right]}{\left[(P_j r+P_j\lambda-\lambda)\exp(rL_j)+\lambda(1-P_j)\right]}\;;\quad j\in(1,n^*). \tag{4}$$

j's demand price at t=0 is also, by symmetric reasoning, his supply price -- where, of course, the demand price is given the standard interpretation of being the maximum price which j is willing to pay for a bond and the supply price that of being the minimum price required to induce j to sell a bond. Thus, if $X_j^S(0)$ refers to j's supply-price at t=0, then we have

$$X_j^S(0) = \left(\frac{r}{X}\right)\frac{\left[\{X(1-m_j)+R\}\exp(\rho(1-m_j)(L_j-T))\right]}{\left[(P_j r+P_j\lambda-\lambda)\exp(rL_j)+\lambda(1-P_j)\right]}\;;\quad j\in(1,n^*). \tag{5}$$

It may be noted that

$$\lim_{P_j\to P_j^*} X_j^D(0) = \lim_{P_j\to P_j^*} X_j^S(0) = X \;;\quad j\in(1,n^*).$$

Individual j's demand-for-bonds function can be represented as in Figure 4 and his supply-of-bonds function as in Figure 5. The horizontal aggregation of the n* individual demand functions will yield a step function which for large n* can be approximated by the smooth function plotted in Figure 6. The market supply function (Figure 7) is likewise derived from the horizontal aggregation of the n* individual supply functions. In plotting the curve in Figures 6 and 7, we have indexed the individuals in descending order of the magnitudes of their demand and supply prices respectively, so that $X^D_{n^*}(0) \geq X^D_{n^*}(0) \geq \cdots X^D(0)$ an'

Figure 4: j's demand for bonds function

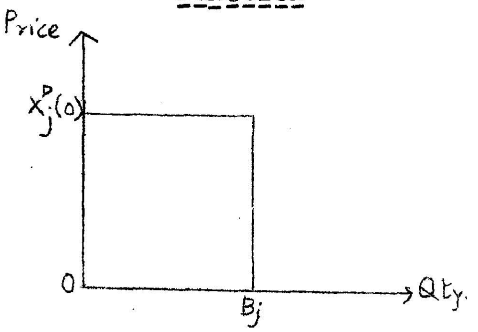


Figure 5: j's supply of bonds function

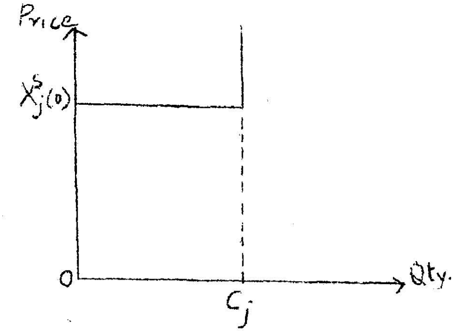


Figure 6: Market Demand Function for Bonds

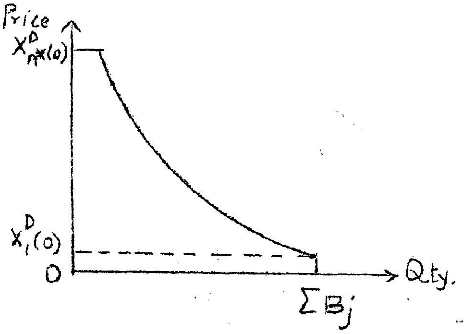
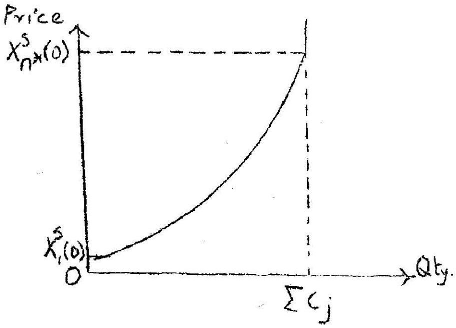


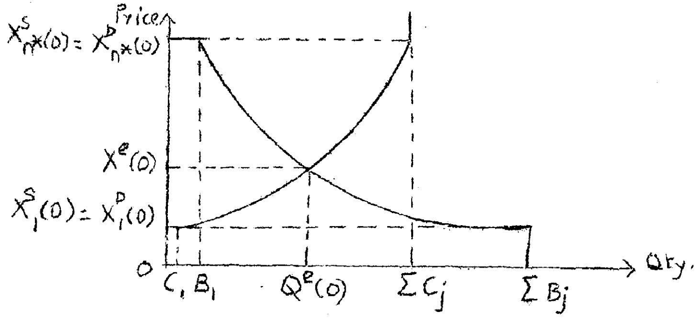
Figure 7: Market Supply Function of Bonds



We are now in a position to obtain a general equilibrium picture of the immediately post-issue market for bonds among black asset-holders. To obtain this picture, we simply superimpose the demand function in Figure (6) on the supply-function in Figure (7), to obtain Figure (8) below. The intersection of the market demand and supply functions determines the equilibrium price ($X^e(0)$) and the equilibrium quantity ($Q^e(0)$) (See Figure (8)) Note that the equilibrium price $X^e(0)$ will be greater than or equal to the issue price X (the latter will be the case if the demand curve intersects the supply curve in its horizontal segment and if, further, $X_1^S(0) = X$). Disregarding this special case, and

as depicted in Figure (8), bonds will sell at a premium in the immediately post-issue market for bonds.[4/]

Figure 8: General Equilibrium in the immediately post-issue market for bonds



III. The Behaviour of the Equilibrium Price of Bonds Over Time

The line of enquiry which is now of interest is that pertaining to the behaviour of the equilibrium price X^e over time. To determine this, we must examine the behaviour of the demand and supply prices of any individual over time. A straightforward generalization of (4) and (5) with respect to time yields the following expressions for the demand and supply prices of the jth individual in time t :

$$(4') \quad X_j^D(t) = \left(\frac{r}{x}\right)\frac{\left[\{x(1-m_j)+R\}\exp(\rho(1-m_j)(L_j-T))\right]}{\left[(P_j r+P_j\lambda-\lambda)\exp(r(L_j-t))+\lambda(1-P_j)\right]} \; ; t\epsilon[0,T); \; j\epsilon D_t$$

$$(5') \quad X_j^S(t) = \left(\frac{r}{x}\right)\frac{\left[\{x(1-m_j)+R\}\exp(\rho(1-m_j)(L_j-T))\right]}{\left[(P_j r+P_j\lambda-\lambda)\exp(r(L_j-t))+\lambda(1-P_j)\right]} \; ; t\epsilon[0,T); \; j\epsilon S_t$$

(It should be noted that after t=0, the community of n* black-marketers is dichotomized into two groups -- one group possessing only bonds and the other group possessing only black assets. Consequently, $X_j^D(t)$ pertains to $j\epsilon D_t$ while

$X_j^S(t)$ pertains to $j \in S_t$, where D_t and S_t are two distinct sets, representing respectively the set of demanders of bonds and the set of suppliers of bonds. It should be remembered that a demander of bonds in one period is a potential supplier in the next period, and <u>vice versa</u>).

It is now a routine matter to verify that

$$(6) \quad \frac{dX_j^D(t)}{dt} = \left(\frac{r^2}{X}\right) \frac{\left[\{X(1-m_j)+R\}(P_j r + P_j\lambda - \lambda)\exp(\rho(1-m_j)(L_j-T)+r(L_j-t))\right]}{\left[(P_j r + P_j\lambda - \lambda)\exp(r(L_j-t)) + \lambda(1-P_j)\right]^2};$$

$$> 0 \text{ for } r > 0 \qquad (t \in [0,T);\ j \in D_t)$$

and

$$(7) \quad \frac{dX_j^S(t)}{dt} = \left(\frac{r^2}{X}\right) \frac{\left[\{X(1-m_j)+R\}(P_j r + P_j\lambda - \lambda)\exp(\rho(1-m_j)(L_j-T)+r(L_j-t))\right]}{\left[(P_j r + P_j\lambda - \lambda)\exp(r(L_j-t)) + \lambda(1-P_j)\right]^2};$$

$$> 0 \text{ for } r > 0 \qquad (t \in [0,T);\ j \in S_t)$$

Ignoring the special (and improbable) case of $r = 0$ ensures that $\frac{dX_j^D(t)}{dt}, \frac{dX_j^S(t)}{dt} > 0$. In other words, each individual's supply and demand prices will rise with time, with the result that the equilibrium price X^e will also increase over time.[5/]

The reason for the tendency of bond-prices to increase over time can be intuitively identified from a consideration of the expression for individual j's demand (supply) price at time t (see equations (4') and (5'). The numerator on the RHS of (4') and (5') refers to the terminal value of investment in a bond, which is the same whenever the investment is made, and the denominator refers to the expected terminal value of holding a rupee's worth of black assets. Since over time the numerator remains constant and the denominator declines (by virtue of the fact that investment in black assets will have fewer years in which to compound the nearer T it is made), the value of the bond increases with time.

This marks a difference in detail from Bharathan's analysis in which $X^e(t)$ is a declining function of time. The reason adduced for the latter type of behaviour of $X^e(t)$ is that with time, the bond offers protection for a lesser period, so that its value declines. Formally, the argument entails placing some restriction on the manner in which the joint probability, at any given point of time, of being undiscovered in all preceding time-periods upto and including the point of time under consideration, behaves with time. In our own analysis, we have made the assumption that this joint probability declines at a constant rate over time : this assumption ensures that $P^u_{t,j} = P_j$, $t \in [-v_j, L_j)$.

Now quite clearly one cannot be dogmatic about this assumption: there is little, on *a priori* grounds, to support the view that any one of the following three assumptions is more plausible than the other two: (i) that the joint probability of being undiscovered declines at a constant rate with time; (ii) that the joint probability declines at a declining rate; and (iii) that the joint probability declines at an increasing rate. Under the first of these assumptions, we would have (as already noted): $\frac{dP^u_j(t)}{dt} = 0$, so that $P^u_{j,t} = P_j \; \forall t \in [-v_j, L_j)$;

under the second assumption, we would have $\frac{dP^u_j(t)}{dt} < 0$;

and under the third assumption we would have $\frac{dP^u_j(t)}{dt} > 0$.

Under assumption (i), as we have seen, the value of the bond increases over time -- and this must be the case, *a priori*, under assumption (ii). Assumption (iii) -- viz. $\frac{dP^u_{j,t}}{dt} > 0$ -- is a necessary condition for the bond-price to decline over time, although it is not sufficient. For note that over time we have two counteracting effects on the price of bonds. The first (which we have already seen) is a positive effect that makes investment in bonds relatively more attractive than investment in black assets, since the compounding effect of investment in black assets becomes smaller as the number of years left to maturity of the bond declines. The second effect

is a negative one wrought by an increasing $P^u_j(t)$ value as t increases : the 'insurance' value of the bond declines with time. If the rate at which the subjective probability estimate of remaining undiscovered increases fast enough (condition (9) below), then the net effect on the price of bonds will be negative. Note that if P^u_j is made time-dependent, then the expression for the demand (respectively) supply price of bonds in time (see equations (4'), (5') will be given by

$$(8) \quad X^D_j(t) \ (\text{resp.}\ X^S_j(t)) = \left(\frac{r}{X}\right) \frac{[\{X(1-m_j)+R\}\exp(\rho(1-m_j)(L_j-T))]}{[(rP^u_j(t)+\lambda P^u_j(t)-\lambda)\exp(r(L_j-t))+\lambda(1-P^u_j(t))]};$$

$$j \in D_t \ (\text{resp. } j \in S_t); \quad t \in [0,T).$$

The equilibrium price of bonds will decline if $\frac{dX^D_j(t)}{dt}, \frac{dX^S_j(t)}{dt} < 0.$

It is readily verified that

$$(9) \quad \frac{dX^D_j(t)}{dt} \ \left(\text{resp. } \frac{dX^S_j(t)}{dt}\right) < 0 \longleftrightarrow$$

$$\text{(i)} \quad \frac{dP^u_{j,t}}{dt} > 0 \quad \underline{\text{and}}$$

$$\text{(ii)} \quad \frac{dP^u_j(t)}{dt} > \frac{r(rP^u_j(t)+\lambda P^u_j(t)-\lambda)\exp(r(L_j-t))}{(r+\lambda)\{\exp(r(L_j-t))-\lambda\}};$$

$$j \in D_t \ (\text{resp. } j \in S_t), \quad t \in [0,T).$$

Briefly, then, while in our analysis we have settled for the assumption that $\frac{dP^u_j(t)}{dt} = 0$, which ensures that $\frac{dX^e(t)}{dt} > 0$, one can have an equally plausible (or abitrary assumption) to the effect that $\frac{dP^u_j(t)}{dt} < 0$ which, together with some additional assumptions on the magnitude of $\frac{dP^u_j(t)}{dt}$ will ensure that $\frac{dX^e(t)}{dt} < 0$. The latter assumptions appears to have been

implicitly made by Bharathan. At a formal level, this is a matter of detail, but it will be seen later that if the Bonds Scheme can result in an injection of black money into the economy in a regime where $\frac{dX^e(t)}{dt} > 0$, then this consequence will hold *a fortiori* in a regime in which $\frac{dX^e(t)}{dt} < 0$.

We can proceed now to introduce the 'white demanders' into our analysis.

IV. The Demand for Bonds Function of the White Demanders

We turn our attention now to the second category of demanders of bonds -- the 'white demanders' alluded to in the introduction to this paper. This category is constituted by a set M of white asset-holders who shall be designated by the running index $g = N+1, N+2, \ldots, N+m$. Assume that individual g has a stock A_g of white money at the time of issue of bonds. There are one of two things he can do with his stock of money : one, he can convert his entire stock into bonds; or two, he can pay tax on his stock of money income, invest the after-tax proceeds in some 'legal' asset (such as a corporate share, or a fixed deposit account in a bank, or whatever), pay tax on the return from investment, reinvest the accumulated after-tax capital, and so on, from period to period till the time (L_g) which is of economic relevance to him. This is what Bharathan refers to as the 'legal route'. If g decides to invest his entire assets in bonds, the terminal value of his investment will be given by

$$(10) \qquad V_g' = \frac{A_g}{X}\{X(1-m_g)+R\}\exp(\rho(1-m_g)(L_g-T)), \quad g \in M.$$

On the other hand, if he decides to adopt the 'legal route', the terminal value of his investment will be given by

$$(11) \qquad W_g' = A_g(1-m_g)\exp(\rho(1-m_g)L_g), \qquad g \in M.$$

Clearly, g will invest in bonds only if $V_g' \gtrless W_g'$. Noting now that according to the terms of the Bonds Scheme, X = Rs.10,000/-, R = Rs.2000/- and T = 10 years, and assuming $\rho = 0.1$, it can be verified that the value of m_g for which V_g' and W_g' are equal is 0.59 : all those individuals whose marginal tax-rates equal or exceed 0.59 will find it worthwhile to purchase bonds at the time they are issued. If, however, the return on investment in the 'legal route' is higher at 20 per cent (i.e. if $\rho = 0.2$), then the marginal tax-rate at which an individual will become indifferent between bonds and white assets is 0.72; since the maximum legal marginal tax-rate is 0.70, nobody will invest in bonds at the time of their issue. Briefly, it is more than likely that at the time of issue of bonds there will be no demand for them -- or putting it differently, that the maximum price any individual will be willing to pay for a bond will be less than X.

This brings us to the question of deriving individual g's demand-price for bonds. The maximum price (call it $X_g^D(t)$) which individual g will be willing to pay for a bond at time t is clearly that amount which, if he were to invest it in the 'legal route', would yield him the same terminal value as would the bond. $X_g^D(t)$, in other words, is obtained as the solution to the equation

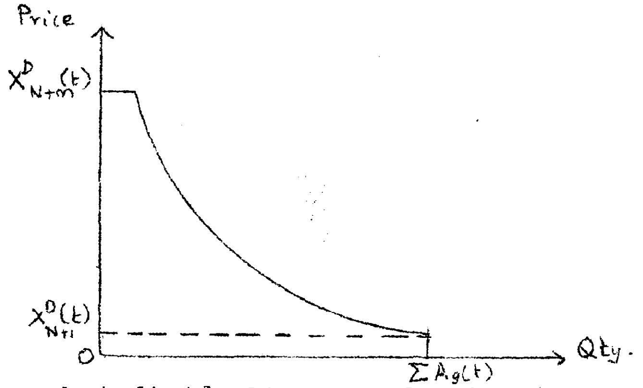
$$X_g^D(t)(1-m_g)\exp(\rho(1-m_g)(L_g-t)) = \{X(1-m_g)+R\}\exp(\rho(1-m_g)(L_g-T)),$$

from which we obtain

$$(12)\quad X_g^D(t) = \frac{\{X(1-m_g)+R\}\exp(\rho(1-m_g)(t-T))}{(1-m_g)}, \quad g\in M;\ t\in(0,T).$$

In accordance with analogous procedures adopted earlier, the demand-for-bonds function for white asset-holders can be represented as in Figure 9 below. (Note that the individuals have been indexed such that $X_{N+m}^D(t) > X_{N+m-1}^D(t) > \cdots > X_{N+1}^D(t)$).

Figure 9 : Market Demand-for-Bonds function of White Demanders



Note finally that each individual's demand-price increases with time, as manifested in

$$(13)\quad \frac{dx^D_g(t)}{dt} = \frac{\rho(1-m_g)\{X(1-m_g)+R\}\exp(\rho(1-m_g)(t-T))}{(1-m_g)}$$

$$> 0 \quad \text{for } \rho > 0 \qquad (g \in M;\ t \in (0,T)).$$

V. The Effect of the Activities of the White Demanders on the Equilibrium Price as Determined by the Black Demanders

How do the activities of the second category of demanders of bonds affect the bond market? Do their activities affect the equilibrium price in the bond market? (Note that if this is the case, it will be so only after t=0, for at t=0 white money-holders will satisfy their demand, if any, directly by purchase of bonds from the Government when the bonds are being issued). At any time t (>0), the equilibrium price in the bond-market will be affected by the demand for bonds from the white demanders only if $X^D_{N+m}(t) > X^e(t)$ --- where, of course, X^D_{N+m} is the maximum demand price of the prices of the various white demanders, and X^e is the equilibrium bond-price as determined by the black demanders. Representing the black demanders' market demand curve for bonds in time t by $D_B(t)$ and the horizontal aggregation of the market demand-curves of both black and white demanders by $D_{B+W}(t)$, we could have two situations, as depicted in Figures (10a) and (10b).

Figure (10a) clearly demonstrates that the equilibrium price in the bond-market, as determined by the activities of black demanders, is unaffected if $X^D_{N+m}(t) \leqq X^e(t)$; Figure (10b) however depicts a situation in which $X^D_{N+m}(t) > X^e(t)$ --- and in this situation, the market clears at a price $X^{e*}(t)\ (> X^e(t))$. The quantity of white assets converted into bonds is then given by $\Delta Q^e(t) \equiv Q^{e'}(t) - Q^{e*}(t)$, where $Q^{e'}(t)$ is the quantity of

Figure (10a): General Equilibrium in which White Demanders are Ineffective

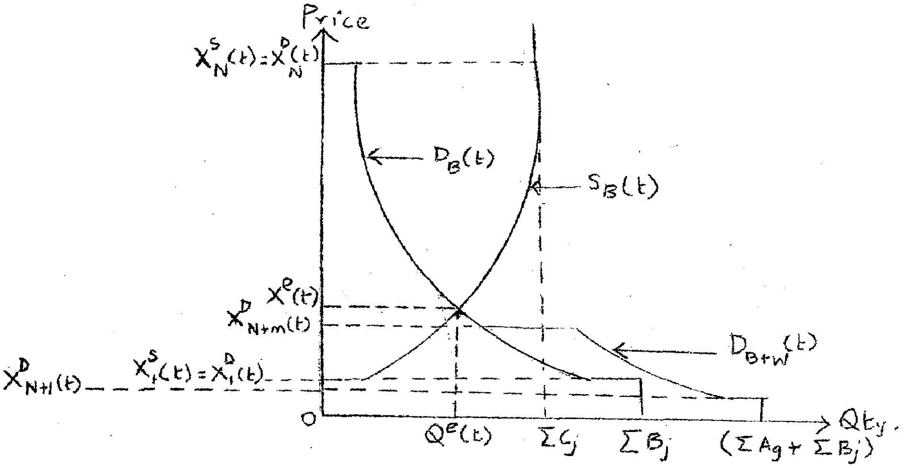
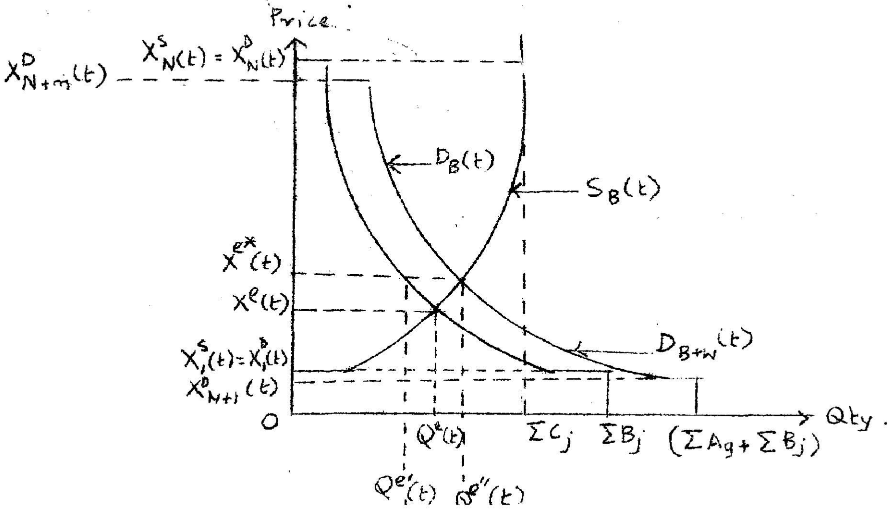


Figure (10b): Equilibirum in which White Demanders are Effective



bonds purchased by black demanders and $Q^{e*}(t)$ is the total equilibrium quantity of bonds transacted in the market (see Figure (10b). $\Delta Q^{e}(t)$ then represents an injection of black money into the economy at time t : it is the quantity of white assets that has been converted into black in exchange for bonds. Now let $T' \equiv \text{Min}\ (T, T^{o})$, where T^{o} is the time at which all bonds possessed by the black asset-holders enter into the hands of the white demanders. Then, a general expression for the total injection of black money into the system is furnished by $\int_{1}^{T'} \Delta Q^{e}(t)\, dt$. The net injection of black money into the system will be given by the gross injection less the stock of black assets which was initially converted into bonds by black demanders, viz., $\sum_{1}^{n^*} C_j$. An expression for the net injection of black money into the system is then furnished by

$$(14) \quad G = \int_{1}^{T'} \Delta Q^{e}(t)dt - \sum_{1}^{n^*} C_j$$

It is immediately clear that the Special Bearer Bonds Scheme which was designed to reduce the quantum of black money in the economy will only have succeeded in increasing this quantum if $G > 0$! Even if $G < 0$, so long as $\int_{1}^{T'} \Delta Q^{e}(t)dt > 0$, the quantity of black assets converted into bonds at the time of their issue will be an overestimate of the quantity of black assets actually mopped up by the scheme.6/

The preceding analysis then leads to the conclusion of this paper, outlined in the Introduction -- namely, that the initial 'success' of the Scheme would depend on black-marketeers possessing a high subjective probability of being raided by the Tax Authorities; and further, that even if such is the case, the success of the Scheme could be diluted by injections of black money into the system due to the development of a demand for bonds among white money-holders.

It only remains to examine some underlying assumptions of the model.

VI. Two Assumptions of the Model

In this section, two particularly restrictive assumptions that have been employed in the preceding analysis are discussed briefly.

(a) The first of the two tricky assumptions employed in the model is that the uncertainty confronted by a black-marketer is essentially an uncertainty regarding which of only two states of nature can occur : one in which the black-marketer is discovered with all his black assets in his possession; and another in which he is not caught at all. In principle, one should allow for an individual being caught in possession of an entire continuum of proportions from zero to one of his stock of black assets in each time period. When this possibility is allowed for, the expression one obtains for individual j's expected terminal value, at the time of issue of bonds, from retaining his black assets as black assets becomes a very complicated one. Use of a 'pay-off' tree should facilitate confirmation of the fact that this expression is given by

$$(15)\quad \overline{W_j} =$$

$$A_j \int_{\theta_{L_j} \in (\frac{1}{1+\lambda}, 1)} \left[\{1 - P_j(\theta_{L_j})\}\{1 - \lambda(1 - \theta_{L_j})\} \right] d\theta_{L_j} \exp(r L_j)$$

$$- \lambda A_j \left[\int_{t \in (0, L_j)} \left\{ \int_{\theta_t \in (\frac{1}{1+\lambda}, 1)} \int_{\theta_{t-1} \in (0, \frac{1}{1+\lambda})} \cdots \int_{\theta_{-v_j} \in (0, \frac{1}{1+\lambda})} \prod_{\tau \in (-v_j, t)} (1 - (1+\lambda)\theta_\tau) P_j(\theta_t)\, d\theta_t \cdots d\theta_{-v_j} \right\} \times \exp(rt)\, dt \right]$$

where $P_j(\theta_t)$ is the probability estimate of individual j that he will be discovered with a proportion θ_t of black assets in his possession in time t.

Having stated this, it must be conceded that the present writer has been unable to do the appropriate probability-modelling required to generate the kinds of comparitive statics results that have been obtained for the two-states-of-nature case. The consequences of relaxing the latter assumption are left as a mathematical modelling problem the solution of which has eluded the present writer's grasp.

(b) A second aspect of the model that might be unpalatable is its requirement that a black demander should hold his entire assets either in the form of bonds or of black assets, never in the form of a <u>combination</u> of the two. To employ the language of James Tobin (1958), the agent in our model is a 'plunger' not a 'diversifier'. The reason for this is easy to locate. Consider the decision problem of the black demander. Let α_j be the proportion of black assets (whose stock at time t=0 is A_j) that individual j decides to retain <u>as</u> black assets, and so $(1-\alpha_j)$ the proportion which he decides to invest in bonds. His expected return (in terms of terminal value) from allocation of his stock of assets between black assets and bonds in the proportions $(\alpha_j, (1-\alpha_j))$ will then by given by $\bar{Y}_j\,[\equiv \alpha_j \bar{W}_j + (1-\alpha_j) V_j]$, and the variance of this return will be given by $\sigma_y^2\,[\equiv \alpha_j^2 A_j^2 \{P_j (\exp(rL_j))^2 - \lambda^2 (1-P_j) \times \int_0^{L_j} (\exp(rt))^2 dt\} - (\alpha_j \bar{W}_j + (1-\alpha_j) V_j)^2]$. Individual j's decision problem can now be seen as that of choosing α_j in such a way as to maximize his von Neumenn Morgenstern expected utility function U_j; using the standard Taylor's approximation formula, his decision-problem can be written down formally as:

$$\underset{\alpha_j}{\text{Maximize}} \quad U_j = u_j(\bar{Y}_j) + \frac{1}{2} u_j''(\bar{Y}_j)\sigma_y^2$$

$$\text{Subject to} \quad 0 \leqq \alpha_j \leqq 1$$

Where $u_j(.)$ is a utility function defined over the return that obtains for each possible state of nature.

Now what we have done in the entire preceding analysis has been to specialize the utility function $u_j(\bar{Y}_j)$ to $\bar{Y}_j$: the utility function has been taken to be represented by the expected return from holding α_j proportion of assets <u>as</u> black assets, and $(1-\alpha_j)$ proportion as bonds. Notice that $\bar{Y}_j\,[\equiv \alpha_j \bar{W}_j + (1-\alpha_j) V_j]$ is a <u>linear</u> function; in extremizing any linear function, one always obtains a corner solution ($\alpha_j = 1$ or 0), which explains why in our model, an individual will possess all of one kind of asset and none of the other.

Relaxing the assumption of a linear utility function $u_j(.)$ would entail relaxing the assumption of risk-neutrality on the part of the agent; with a strictly concave utility-function, the agent would be risk-averse. Risk-aversion would, in general, entail asset-diversification. Further, if bonds are perceived as net wealth (on this see Robert Barro, 1974), and if certain reasonable (Arrow (1965)-type) restrictions on the nature of risk-aversion are placed, than this will have implications for the fraction of one's portfolio which one allocates to risky assets when the size of wealth increases and the portfolio is a combination of some risky asset(s) and a safe asset.[7/] This clearly has consequences for the price of bonds over time. For reasons of expository simplicity, we have worked within a relatively impoverished framework of risk-neutrality.

(c) A final point[8/] may be noted: the model presented in this paper is a highly stylized one; none of the equations are estimating equations, suitable for econometric investigation.

Concluding Remarks

The Special Bearer Bonds Scheme is a very remarkable one; through a peculiar accounting procedure it has transformed what should, from the Government's point of view, be an Accounts Receivable (the tax-evasions of black marketers) into an Accounts Payable (the issue of debt in the form of bonds). The debt must be serviced -- presumably by raising revenue through taxes or by cutting Government spending; in either event, legitimate tax-payers might be expected to suffer. This is not just a case of robbing Peter to pay Paul, but a singular one of robbing Peter to pay Paul who has already robbed the Government.

Quite apart from this, as we have seen with the aid of a simple supply-demand model built on Bharathan's earlier analysis, it could well be that the Bonds Scheme is self-defeating, and

will end up with a net injection of black money into the economy. This is a discouraging enough comment on a scheme which, in addition, might find it difficult to countenance an unfussy scrutiny from the point of view of its ethical implications.

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Notes

1/ Both the debt to and the liberties taken with Bharathan's analysis should become apparent to the reader as he proceeds.

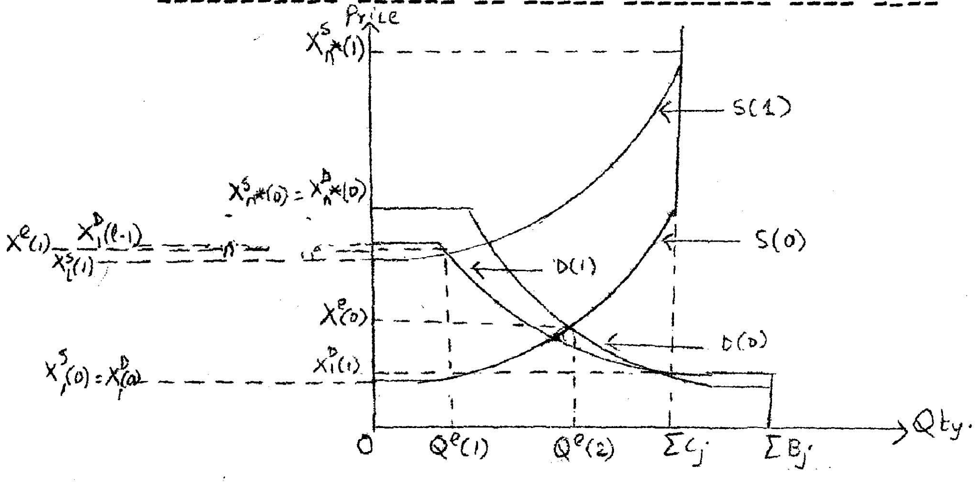
2/ Notice that we always require $P_j' \geq \frac{\lambda\{\exp(\gamma L_j)-1\}}{(\gamma+\lambda)\exp(\gamma L_j)-\lambda}$ as otherwise $\bar{W}_j'$ would be negative, and there would be no point in j holding any black assets at all.

3/ A brief digression is in order here. To be strictly accurate, the maximum price we have derived is the maximum price j would be willing to pay at the time of issue of bonds. Note that the actual price he has paid is only X. Thus, given that he has purchased a total of C_j/X bonds, the consumer's surplus he has earned on these bonds is given by $(X_j^D(0)-X)\cdot C_j/X$. Consequently, with an unfulfilled demand for bonds of B_j, the maximum price j will be willing to pay for a bond, once the issue of bonds has stopped, must, in view of the consumer's surplus he has earned on bonds already purchased, be somewhat more than $X_j^D(0)$. In the text we shall completely ignore the problem of reckoning the consumer's surplus accruing to buyers and the 'producer's surplus' accruing to sellers of bonds. These surpluses can be taken into account in determining the demand and supply-prices over time -- but only at the expense of employing a good deal of messy algebra which has no particular substantive content.

4/ Here a small point of disagreement with one aspect of Bharathan's analysis may be registered. He seems to suggest that the emergence of a premium in the bond market could in itself spell an injection of black money into the system, by virtue of the premium-bearing bond laying claim on resources for which the bond is being exchanged. The disagreement on this score stems from the consideration that the opportunity use of the resource which is exchanged for the premium-bearing bond would in any event have been only in the black market : the only consequence of the exchange is a transfer of ownership of black assets from one black-marketeer to another -- without, therefore, any question of an injection of black assets.

5/ This phenomenon can be captured in the figure drawn below which reflects the underlying comparitive statics. Assuming for convenience that trading occurs at the beginning of each time period, we can see how the market-supply and demand curves look at two successive points in time, say t=0 and t=1. (See figure below).

Comparitive Staticsof Price Behaviour Over Time



In the figure above, all prices and quantities are designated by the figure in brackets following them, according to the point

in time (t=0 or t=1) to which they relate. Thus $X^e(o)$, $Q^e(o)$ relate to the equilibrium price and equilibrium quantity at t=0. Now let l be the integer for which $X_l^D(0) = X^e(0)$ $(= X_l^S(0))$. It is then clear straightaway that the individuals who will have sold bonds at t=0 are individuals 1, 2 ..., l-1, and the individuals who will have purchased bonds at t=0 are individuals l, l+1, ..., n*. What happens to the market supply and demand curves at t=1? Note first that individuals 1, 2, ..., l-1 do not possess any bonds at all at t=1, having sold them all at t=0. Consequently, the supply of bonds from individuals 1, 2, ..., l-1 will be perfectly inelastic at zero quantity of bonds at t=1, while the supply of a larger quantity of bonds from each of the individuals l, l+1, ..., n* will become perfectly elastic at a higher price at t=1 than at t=0, since $\frac{dX_j^S(t)}{dt} > 0$. Briefly, the market supply curve will shift from S(0) to S(1) (see figure). What of the market demand curve? Again note that individuals l, l+1, ..., n* do not possess any black assets at t=1, having traded these for bonds at t=0. Consequently, at t=1, the demand for bonds from individuals l, l+1, ..., n* will be perfectly inelastic at zero quantity of bonds. The demand curves of individuals 1, 2, ..., l-1 will each, however, become elastic at a higher price at t=1 than at t=0, since $\frac{dX_j^D(t)}{dt} > 0$. The market demand curve's shift will then be from D(0) to D(1), as pictured in the figure. The intersection of S(1) and D(1) furnishes the equilibrium price $X^e(1)$ $(> X^e(0))$ and the equilibrium quantity $Q^e(1)$. It is a simple enough matter to picture all subsequent shifts in the supply and demand functions over time.

6/ How likely is it that a substantial quantum of black money will be injected into the economy due to the activities of the white demanders? This is a matter

that can only be verified empirically: in particular, it will depend on the values of the parameters m_j, P_j, λ, τ, ρ, etc., and the distribution of the individual specific parameters across individuals. One thing that can be said though is that the model developed in this paper will less readily support the possibility of a substantial injection of black money into the system than Bharathan's analysis would seem to. The reason for this is a departure in detail from Bharathan's analysis : namely that though the demand prices of the white demanders in both Bharathan's analysis and this one will increase with time, the market-clearing price of bonds as determined by black demanders, X^e, will in our model <u>increase</u> with time but in Bharathan's scheme, <u>decline</u> with time. The relevance of the direction of slope of the $X^e(t)$ function to the plausibility of injection of black money into the system is brought out in the following. Suppose for a moment that the white demanders are not effective to begin with (i.e. suppose Figure (10a) depicts the initial situation we are talking of). Then, even though the demand price of each white demander is increasing with time at an increasing rate, in terms of this model, so is the equilibrium price $X^e(t)$ which is determined by the black demanders of bonds. If the equilibrium price $X^e(t)$ rises at a faster rate than the maximum demand price of the white demanders, the latter will never catch up with the former; the white demanders will never become effective, and it will be the case that $\int_1^T \Delta Q^e(t)dt = 0$: there will be no injection of black money. Even if the maximum demand price of the white demanders does catch up with X^e at some time, say t_0, this will have the effect of raising X^e even more than otherwise in the next period: there is no guarantee that the 'white' demand price will subsequently (i.e. after t_0) catch up with X^e. In Bharathan's analysis, with the

(white) demand price increasing with time and the (black) equilibrium price declining with time, conditions are more favourable for the demand-price catching up with the equilibrium price; once the former has caught up with the latter, the demand price will remain above the equilibrium price with the gap between the two widening.

7/ For example, with declining absolute risk-aversion, the fraction allocated to the risky asset(s) would increase.

8/ One additional point might require clarification. It might be imagined that an important assumption underlying the model is that the stock of black assets possessed by individual j at the time of issue of bonds is in the nature of a stock of fixed capital which does not vary in size in the short-run. This might be a valid enough assumption if "black assets" are interpreted to include only such assets as land or jewellery, the supply of which is relatively inelastic in the short-run. However, a part of "black assets" could also well be constituted by inventories of commodities -- such as, say, vegetable oil or foodgrains or whatever -- and such assets would certainly be subject to accretions over time. Is there then any systematic way of accommodating these accretions into our model, which is permised on the notion of a stock A_j of unvarying size? In principle, yes : it would require invoking two assumptions -- those of a perfect capital market and perfect foresight -- and it would require resorting to the simple expedient of converting any flow into a stock *via* the technique of discounting. To be specific, let $a_j(t)$ represent the net addition of 'black' inventories of commodities (expressed in value terms) to the stock of black assets possessed by individual j, at time t ($t \in [0, L_j)$). Assuming perfect foresight ensures that all the $a_j(t)$ will be known in advance (i.e. at the time of issue of bonds). The present value of the flow of expected (which is the same here as 'known') accretions to j's stock of black

assets is than given by $K_j = \int_1^{t_j'} a_j(t)\exp(-rt)dt$. We can now redefine A_j as $A_j' \equiv C_j+B_j+K_j$ where, as before C_j refers to the stock of liquid black assets, B_j to the stock of illiquid assets and K_j to the discounted value of future (known) accretions to the stock of black assets. Once the issue of bonds has terminated, the unfulfilled demand for bonds at t=0 will be given by (B_j+K_j). In the market for bonds which subsequently develops, payment for bonds can be assumed to be effected with (a) illiquid assets (barter) and (b) with promissory notes: if a perfect capital market is assumed, it would be possible to say that the future demand for bonds, represented now by the discounted value of the expected accretions to stocks of black assets will be paid for at t=0 in the form of slips of paper containing promises to pay the amount due to the seller of bonds, together with interest at the rate r on the amount due -- these promises to be redeemed as and when the accretions to stocks occur. The model, after being subjected to complication by allowing for accretions to stocks, has now been reduced to precisely the model described earlier.

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